Table 1

x_{SVD}	$\bar{x}$, Eq. (7)	x_{ED}, Eq. (10)	x_{SED}, Eq. (13)
0.5294	0.2952	0.5262	0.5295
0.9995	1.0016	1.0016	0.9995
1.0001	1.0048	1.0048	1.0001
2.1179	2.1626	2.1043	2.1179

where η is a factor to be determined. From Eq. (17) we have

$$\|x_{SVD} - x_{SED}\| \le \frac{\epsilon^2}{\lambda_r(\lambda_r^2 - \epsilon^2)} \|b\| = \frac{\eta^2}{1-\eta^2} \frac{\|b\|}{\lambda_r}$$

In order to guarantee that the solution x_{SED} has the accuracy of 10^{-1}, the η has to satisfy the following equation:

$$\frac{\eta^2}{1-\eta^2} \frac{\|b\|}{\lambda_r} \le 10^{-1}$$

or

$$\eta \le \sqrt{\frac{q}{1+q}} \quad \text{where} \quad 10^{-1} \frac{\lambda_r}{\|b\|} = q$$

Thus, even though $(\bar{x} - \bar{\bar{x}})/2$ is a measure of the accuracy of an SED solution, it is seen that λ_r is required to accurately bound $\|x_{SVD} - x_{SED}\|$.

Numerical Example

To compare SED with the ED approach, the same numerical example as in Ref. 2 is adopted. The coefficient matrix of linear equations

$$Ax = b$$

is a 4×4 singular matrix

$$A = \begin{bmatrix} 1.0596 & 2.1727 & 2.4828 & 4.2386 \\ & 6.5001 & 5.5023 & 8.6910 \\ & & 9.4852 & 9.9307 \\ \text{symmetric} & & & 16.9547 \end{bmatrix}$$

and the RHS of the equations is

$$b = (14.1924 \quad 31.5561 \quad 37.3317 \quad 56.7697)^T$$

The lowest two eigenvalues of the matrix A are

$$\lambda_4 = 0 \qquad \text{and} \qquad \lambda_3 = 1.6046$$

The equations can be very sensitive to small changes in the RHS of the equation, and dramatic errors can result in the solution.[2]

The application of Eqs. (12), (13), and (18) for $\epsilon = 0.05$ indicates that $\eta = 0.03116$,

$$\|x_{SVD} - x_{ED}\| \le 0.0188 \|b\|$$

while

$$\|x_{SVD} - x_{SED}\| \le 0.0006 \|b\|$$

The solutions using SVD, ED, and SED are listed in Table 1 for $\epsilon = 0.05$. It can be observed that more accurate results are obtained by the SED approach rather than ED approach.

Concluding Remarks

The following have been mathematically proven and shown by a numerical example:

1) SVD, as is well known, is a very strong numerical analysis tool for solving ill-conditioned linear equations, but it can be computationally costly when there is only small rank deficiency.

2) As was shown here, ED is a first-order approximation of SVD, and as was shown in Ref. 2, ED is computationally very efficient for large matrices with small rank deficiency, because only the first nonzero and all zero eigenvalues and eigenvectors are required.

3) As has been shown in this Note, SED is a second-order approximation of SVD and only requires the solution of two sets of linear equations. However, unless one obtains the first nonzero eigenvalue, the errors involved will be difficult to bound accurately.

References

[1]Ojalvo, I. U., and Ting, T., "Interpretation and Improved Solution Approach for Ill-Conditioned Linear Equations," *AIAA Journal*, Vol. 28, No. 11, 1990, pp. 1976–1979.

[2]Ojalvo, I. U., "Improved Solution Ill-Conditioned Algebraic Equations by Epsilon Decomposition," *AIAA Journal*, Vol. 29, No. 12, 1991, pp. 2274–2277.

Actuator Placement Optimization by Genetic and Improved Simulated Annealing Algorithms

Junjiro Onoda*
Institute of Space and Astronautical Science, Kanagawa 229, Japan
and
Yoji Hanawa†
University of Tokyo 113, Tokyo, Japan

Introduction

STRINGENT shape accuracy is required for many space structures such as antenna reflectors. An effective approach to improve the shape accuracy of truss structures is to correct the static distortion in orbit by using variable-length active members.[1] The number of actuators is usually limited because of cost and weight, and so it is important to locate the actuators optimally.

This type of optimization problem can be formulated as an integer programming problem.[1] When the number of truss members is large, it is almost impossible to obtain the global-optimal placement because of the discrete nature of available actuator locations and the resulting huge number of possible configurations. Therefore, many approaches[1-3] have been proposed to obtain a near-optimal solution with a reasonable amount of calculation. However, a more efficient and reliable approach seems to be required.

In this Note, the improved simulated annealing approach[4] (ISA) is applied to this type of problem for the first time. A genetic algorithm[3] (GA) is modified into a suitable form for the present problem and also applied. Finally, their performances are compared with simulated annealing[2] (SA), worst-out best-in[1] (WOBI), and exhaustive single-point substitution[1] (ESPS) based on a realistic example.

Received Feb. 24, 1992; presented as Paper 92-2558 at the AIAA/ASME/ASCE/AHS/ASC 33rd Structures, Structural Dynamics, and Materials Conference, Dallas, TX, April 13–15, 1992; revision received Sept. 25, 1992; accepted for publication Sept. 30, 1992.

*Professor.

†Graduate Student, Department of Aeronautics.

Definition of Optimization Problem

In this Note, we seek to optimize actuator placement in truss structures for the correction of static deformation. The distortion is due to a set of length errors of the members e. If we assume a linear system, the residual deformation of controlled structures is[5]

$$\delta \equiv \Psi e + U\theta \equiv \psi + U\theta \quad (1)$$

where e is a vector composed of the length errors of the members, θ is the vector of actuations of the actuators, ψ is the distortion before the correction, and Ψ and U are the matrices that represent these effects, respectively. The optimal value of θ (which minimizes the mean square of residual distortion $\delta^2_{\rm rms}$) and the resulting residual distortion δ can be explicitly obtained as shown in Ref. 5. Usually, ψ can be known only statistically at the design phase. In such a situation, the effectiveness of the shape control is measured by, e.g.,[5]

$$g^2 \equiv E(\delta^2_{\rm rms})/E(\psi^2_{\rm rms}) \quad (2)$$

where E denotes the expectation operator and $\psi^2_{\rm rms}$ the mean square of ψ. The present problem is to obtain the placement of n_a actuators that minimizes g^2.

Economical Calculation of g^2 in the Iteration Process of Optimization Scheme

We assume that the actuators are variable-length truss members integrated into the structure by replacing passive members. Generally, the stiffnesses of active members may be different from those of passive members. Therefore, when the actuator placement varies, Ψ and U vary and have to be recalculated. But neither δ nor g^2 are affected by the actuator stiffnesses, provided that their strokes are great enough. This means that any placement optimization problem is equivalent to the problem in which each passive member is replaced with an active member whose stiffness is identical. So we can solve the equivalent problem instead of the original one, in which Ψ does not need to be recalculated.

Even in the equivalent problem, U varies. However, Ψ includes all of the columns associated with all of the members when all of the truss members have length errors as the cause of distortion. In such a situation, the jth column of U is equal to the column of Ψ associated with the passive member where the jth active member is placed. Therefore, once Ψ is obtained, U can be easily assembled by extracting appropriate columns from Ψ for any actuator placement configuration. Because the re-estimation of Ψ requires the inversion of the stiffness matrix[5] whose dimension is much larger than that of U, we can reduce the calculation cost drastically by solving the equivalent problem.

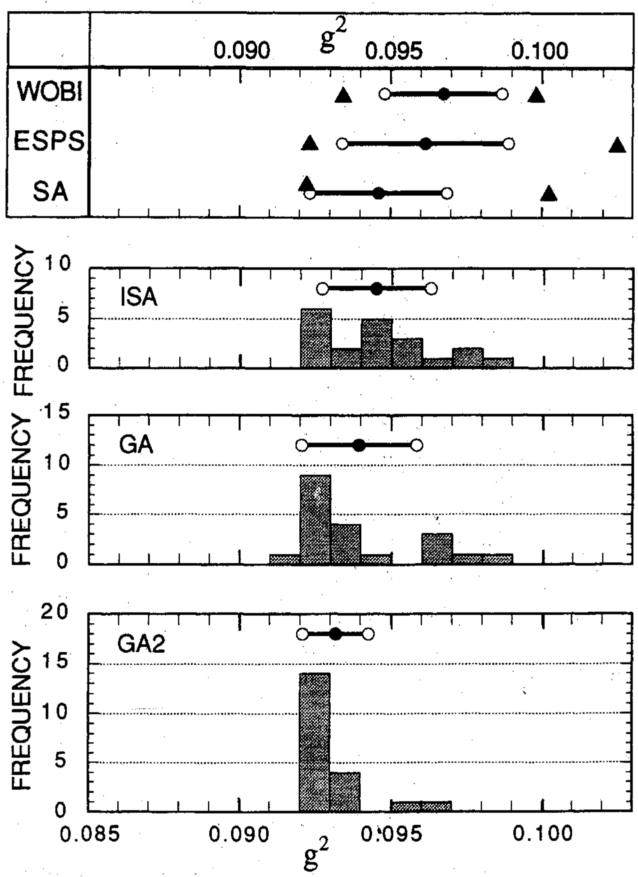

Fig. 1 Histogram, mean values, standard deviations, best and worst values of g^2 of solutions ($M = 2 \times 10^4$).

Genetic Algorithm

GA is a guided random search technique simulating natural evolution. It deals with a population, and the chromosome of each individual represents a candidate solution, i.e., an actuator placement configuration in the present problem. By dealing with this group of solutions, this approach tries to avoid being trapped at local optima. In the standard GA approach,[3] the genes in the chromosome are binary. But, in this study, the representational structure is modified such that the ith gene is an integer that represents the location of the ith actuator. In this richer representational scheme, the number of genes in a chromosome is much smaller than that in the binary representational structure.

The GA approach applied in this study is

1) Create a population by selecting n_g initial configurations and calculate g^2 for each of them.

2) Randomly select a pair of parent configurations under a probability p_p that is a function of the "fitness" (i.e., a monotonically decreasing function of g^2 in this problem) of each configuration.

3) Eliminate the worst configuration (i.e., whose g^2 is the maximum).

4) Create an offspring configuration from the parents by "crossover" and "mutation," accept it as a member of the population, and calculate g^2 for it.

5) If iteration is less than the maximum number, go to step 2.

6) Otherwise, select the best configuration from the current population and stop.

In the present scheme, the crossover operation creates a new chromosome by randomly selecting the genes from the chromosomes of the parents. The mutation operation replaces the value of a randomly selected gene of the offspring with a randomly selected location number under the probability p_m.

In this study, the GA is used in a two-phase manner also. In its first phase, the initial configurations (the individuals of the initial population) for the second phase are generated by GA with a small-size population. The second phase is a GA process with a standard-size population. In this Note, the two-phase GA is abbreviated as GA2.

Improved Simulated Annealing Algorithm

ISA is a kind of combination of SA and GA. Like GA, it deals with a group of solutions. The performance of the group is improved by both the random search similar to that in SA and the selection similar to that in GA. This approach demonstrated better performance for a block placement problem than SA.[4] The scheme is outlined as follows:

1) Select initial placement configurations I_{0i} ($i = 1, \ldots, n_g$), calculate g^2_{0i}, and select an initial value of T.

2) Randomly select a configuration whose g^2 is larger than the average and replace it with a randomly selected configuration whose g^2 is less than the average.

3) For $i = 1, n_g$.

a) Calculate g_{1i}^2 for a configuration I_{1i} obtained by replacing a randomly selected actuator of I_{0i} to a randomly selected location outside I_{0i}.

b) Set $I_{0i} = I_{1i}$ and $g_{0i}^2 = g_{1i}^2$ under a probability of $\min[\exp\{(g_{0i}^2 - g_{1i}^2)/T\}, 1]$.

4) Set $T = T \times r$.

5) If iteration is less than the maximum number, go to step 2.

6) Otherwise, select the best configuration from $I_{0i}(i = 1, \ldots, n_g)$ and stop.

Where $\min[\alpha_1, \alpha_2]$ is the minimum value of α_1 and α_2, T the "temperature" simulating the annealing process, and r the "cooling rate" $(0<r<1)$. Steps 3 are same as SA, and step 2 improves the performance as a group similarly to GA.

Numerical Example

The performance of each approach should be evaluated and compared by using a realistic example. In this study, a parabolic three-ring tetrahedral truss composed of 234 members is investigated. The variation of length is assumed to consist of independent random variation plus correlated random variation, and the ith row jth column element of the covariance matrix of e, Σ, is assumed as

$$\Sigma_{ij} = 1.0 \times 10^{-4}(\delta_{ij} + \gamma_{ij}) \tag{3}$$

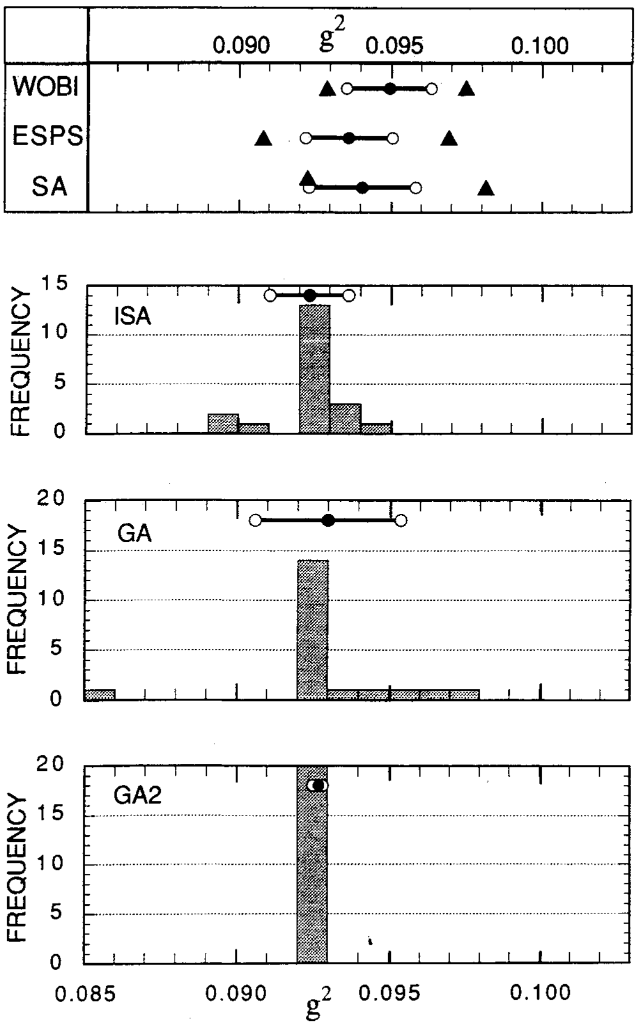

Fig. 2 Histogram, mean values, standard deviations, best and worst values of g^2 of solutions ($M = 1 \times 10^5$).

$$\gamma_{ij} = \begin{cases} 1 & \text{if both } i \text{ and } j \text{ are in the upper surface} \\ 1 & \text{if both } i \text{ and } j \text{ are in the lower surface} \\ 0 & \text{if } i \text{ or } j \text{ is a core member} \\ -1 & \text{otherwise} \end{cases} \tag{4}$$

where δ_{ij} is Kronecker's delta function.

The 37 displacements of all of the upper surface nodes in the direction parallel to the parabola axis are assumed to be sensed. All the truss members are candidate installation locations for the active members. The number of actuators is limited to 10, resulting in 1.1×10^{17} possible placement configurations.

The actuator locations which minimize g^2 are searched by GA, GA2, and ISA. To compare their performances, WOBI, ESPS,[1] and SA[2] are also applied. In order to compare fairly, the allowable number of iterations using each approach (which comprises most of the total calculation) is limited to M. In this note, the results are compared in the cases of $M = 2 \times 10^4$ and 1×10^5.

Results and Discussion

By using roughly optimal control parameters estimated from preliminary investigation, 20 results are obtained by each approach with different initial configurations. Figures 1 (for $M = 2 \times 10^4$) and 2 (for $M = 1 \times 10^5$) show the histograms of the results from GA, GA2, and ISA as well as μ and $\mu \pm \sigma$ of the 20 results from various approaches, where μ is the mean value and σ the standard deviation. The horizontal bars indicate the ranges of $\mu \pm \sigma$. Furthermore, the best and worst solutions are also indicated by the triangular marks for WOBI, ESPS, and SA.

Figure 1 indicates that all the approaches work well for a realistic problem. It can also be seen that GA2 is the best, GA second, and ISA third in the sense of the mean value of solutions. The order of GA and ISA is reversed in the case of standard deviation, although the difference is not large. It is interesting that all the best three approaches deal with a group of solutions.

When $M = 1 \times 10^5$, Fig. 2 illustrates that ISA is the best, GA2 second, and GA third in the sense of average. However, in the sense of standard deviation, GA2 is the best. The figure again demonstrates the better performance of the approaches that deal with a group of solutions. The difference is not large in this case either.

Conclusions

GA, GA2, and ISA are applied to the actuator placement optimization problem for statistical static distortion correction of truss structures. The performances of these approaches are compared based on a three-ring tetrahedral truss example, together with ESPS, WOBI, and SA. So far as investigated here, GA, GA2, and ISA show higher performance than the others.

References

[1]Haftka, R. T., and Adelman, H. M., "Selection of Actuator Locations for Static Shape Control of Large Space Structures by Heuristic Integer Programing," *Computers & Structures,* Vol. 20, No. 1–3, 1985, pp. 575–582.

[2]Kincaid, R. K., "Minimization Distortion in Truss Structures: A Comparison of Simulated Annealing and Tabu Search," AIAA Paper 91-1095, April 1991.

[3]Rao, S. S., Pan, T.-S., and Venkayya, V. B., "Optimal Placement of Actuators in Actively Controlled Structures Using Genetic Algorithms," *AIAA Journal,* Vol. 29, No. 6, June 1991, pp. 942, 943.

[4]Koakutsu, S., Sugai, Y., and Hirata, H., "Block Placement by Improved Simulated Annealing Based on Genetic Algorithm," *Transactions of the Institute of Electronics, Information and Communication Engineers,* Vol. J73-A, No. 1, Jan. 1990, pp. 87–94 (in Japanese).

[5]Burdisso, R. A., and Haftka, R. T., "Statistical Analysis of Static Shape Control in Space Structures," *AIAA Journal,* Vol. 28, No. 8, Aug. 1990, pp. 1504–1508.